# DIFFRACTION OF A PLANE WAVE ON A WEDGE MOVING AT SUPERSONIC SPEED 

# UNDER CONDITIONS OR SPORADIC SHOCK INTERACTION 

PMM Vol. 38, N3, 1974, pp.484-493<br>L. E. PEKUROVSKII and S.M.TER-MINASIANTS<br>(Moscow)<br>(Received August 29, 1973)

The subject of present investigation is the diffraction in such ranges of angles between the fronts of weak incident pressure jump and of an oblique compression shock attached to a wedge of finite opening angle in which the intersection of these occurs at the distorted section of the jump. On the side of smaller angles these ranges border on regions of possible regular interaction between two shocks coming from opposite directions []], while on the side of greater angles these border of regions of shock waves moving in the same direction, which admit uniform streams in the neighborhood of intersection of fronts [2,3]. The resulting boundary value problem has much in common with the similar problem of regular counter-interaction which was considered in [4]. In that paper, as in the present one, the method of analysis is related to the problem of perturbations of a uniform stream behind a plane shock front considered by Lighthill $[5,6]$.

The effect of triple shock configuration is represented by the combined dynamic singularity at the triple point. Some of its properties were predicted by Landau [7].

The particular case of motion of a slender wedge at hypersonic speed was considered by Inger [8, 9]. However his analysis contains statements which contradict existing concepts.

1. The investigated flow. A wedge is moving at supersonic speed $M_{\infty} a_{\infty}$ through a quiescent perfect gas, generating an attached plane compression shock. A weak plane pressure jump, whose front is parallel to the edge of the wedge, moves through the gas toward it at velocity $a_{\infty}$ equal to the speed of sound in the gas. The particle of gas which at the instant of encounter lies at the edge is the center $E$ of the Mach circle whose arcs together with the wedge and the oblique shock constitute the boundary of the


Fig. 1 region of diffraction origination. This region is shown in Fig. 1. The motion is self-similar. The generated flow regions and parameter subscripts used here are the same as in Fig. 1 in [4]. The angles of inclination $\alpha$ of the shock and $\beta$ of the wedge, and angle $\chi$ between the plane of the incident front and the wedge plane of symmetry are assumed to be finite, and the ratio $\varepsilon$ of pressure of the incident shock and that of the quiescent gas is assumed to be a small parameter. The velo-
city of point $E$ relative to the wedge is $a_{1} M$. Axes $x$ and $y$ of the self-similar coordinates have their origin at that point and are, respectively, perpendicular and parallel to the unperturbed compression shock in a plane perpendicular to the wedge edge.

The diffraction modes in which intersection of wave fronts (point $G$ ) occurs within the distorted section of the compression shock are considered. The motion of gas behind the oblique shock is at subsonic velocity relative to point $G$; emergence of any refracted wave front from that point into the region of the nonuniform stream is impossible, only a tangential discontinuity connects it with the diffraction center $E$; perturbations induced by point $G$ propagate over the whole of the diffraction region. The laws of conservation make it impossible to surround the


Fig. 2 triple point by uniform flow regions with any a priori specified accuracy even in any arbitrarily small neighborhood of it, except in cases in which the pressure ratio $p_{1} / p_{o}$ and the angle $\varphi=\chi-\alpha$ berween the shock fronts are bounded by the conditions of bifurcation (existence of a triple shock configuration which separates uniform streams) [10, 11].

The limits of variation for the three input parameters $\beta, M_{\infty}$ and $\chi$, which correspond to the investigated modes can be specified by a set of plane curves. It is, however, possible to define these in terms of the dependence between two parameters represented by the input $M_{c}=M_{\infty} \sin \alpha$ and $\varphi$. When $M_{c}$ and $\varphi$ are known, the position of point $G$ on the unperturbed shock front is readily
determined

$$
\begin{equation*}
y_{G}=-a_{\infty}\left(M_{c} \cos \varphi+1\right) / a_{1} \sin \varphi \tag{1.1}
\end{equation*}
$$

Then the indicated boundary is defined by the geometric locus of points $p_{\infty} / p$, and $\varphi$ which correspond to the coincidence of point $G$ and points $A$ and $C$, where $y_{G}=$ $\mp \sqrt{1-m^{2}}, m=M \sin \gamma$ and $\gamma=\alpha-\beta$. It is represented in Fig. 2 by the outer continuous heavy lines, while the middle corresponds to point $G$ lying at the middle of the perturbed section of the compression shock. Points of the thin continuous lines correspond to bifurcation conditions [10,11]. The meaning of dashed lines is explained in Sect. 5 below.

The flow parameters in regions 1 and 2 are defined by formulas (1.1) and (1.2) appearing in [4]. The dimensionless perturbations of pressure, density, velocity components and the dimensionless coordinates

$$
p=\frac{p^{\prime}}{\rho_{1} a_{1}^{2}}, \rho=\frac{\rho^{\prime}}{\rho_{1}}, u=\frac{u^{\prime}}{a_{1}}, v=\frac{v^{\prime}}{a_{1}}, x=\frac{x^{\prime}}{a_{1} t}, y=\frac{y^{\prime}}{a_{1} t}
$$

are formed from related dimensional quantities $p^{\prime}, \rho^{\prime}, u^{\prime}, v^{\prime}, x^{\prime}, y^{\prime}$ and time $t$.
It was shown in [4] that, after Busemann's transformation $r=2 R /\left(1+R^{2}\right), x=$ $r \cos \theta, y=r \sin \theta$, function $p$ satisfies the Laplace equation within the perturbed
region, the condition $\partial p / \partial n=0$ at the wall, and along arcs of the Mach circle condition $\partial p$ / $\partial s=0$, where $n$ and $s$ are coordinates taken along the external normal and tangent to the contour, respectively.
2. Boundary condition the thock front. The triple point at the distorted section of the shock front defined by the equation $x=m+f(y)$, where $f(y)$ is of the order of $\varepsilon$, separates it into two parts ahead of which the states of gas are different. Linearization of the laws of conservation yields boundary values of functions $u$, $v$ and $p$, which along section $C G$ contain additional terms in comparison with section GA. It is more convenient, however, to define these boundary values for the whole of the shock wave front distorted section $A C$ by the single expression

$$
\begin{align*}
& u=\frac{2}{x+1}\left(1+M_{c}^{-2}\right)\left(f-y f^{\prime}\right)+h_{u} \vartheta\left(y-y_{G}\right)  \tag{2.1}\\
& v=-M_{1} f^{\prime}+h_{v} \vartheta\left(y-y_{G}\right), \quad M_{1}=2 a_{\infty}\left(M_{c}-M_{c}^{-1}\right) /(x+1) a_{1} \\
& p=\frac{4}{x+1} \frac{a_{\infty} \rho_{\infty}}{a_{1} \rho_{1}} M_{c}\left(f-y f^{\prime}\right)+h_{\mathrm{p}} \vartheta\left(y-y_{G}\right) \\
& \vartheta\left(y-y_{G}\right)=\left\{\begin{array}{l}
0, y<y_{G} \\
1, y>y_{G}
\end{array}\right.
\end{align*}
$$

where $x$ is the polytropic exponent and the quantities $h_{u}, h_{v}$ and $h_{p}$ are the same as the second terms in the right-hand sides of formulas (2.2) in [4] (*).

Using Eqs. (2.1) from [4] and eliminating $u$ and $v$ by differentiation along the front image, we obtain boundary conditions for $p$ and for $\partial v / \partial y$ the expression

$$
\begin{align*}
& \left(m^{2}-1\right) \frac{\partial p}{\partial x}+\left[(A+m) y-\frac{m B}{y}\right] \frac{\partial p}{\partial y}=\left(\frac{m B}{y}-A y\right) S(y) \delta\left(y-y_{G}\right)  \tag{2.2}\\
& \frac{\partial v}{\partial y}=\frac{B}{y} \frac{\partial p}{\partial y}-\left(\frac{h_{p} B}{y}-h_{v}\right) \delta\left(y-y_{G}\right)  \tag{2.3}\\
& S(y)=y\left(h_{u} y-m h_{v}\right) /\left(A y^{2}-m B\right)-h_{p} \tag{2.4}
\end{align*}
$$

where $\delta\left(y-y_{G}\right)$ is the Dirac delta function. Consequently, $y$ in the right-hand sides of (2.2)-(2.4) relates to point $G$. Constants $A$ and $B$ are defined by formulas (3.1) in [4].

The Busemann transformation convetrs the distorted section of the shock front $r=$ $m$ sec $\theta$ into an arc of circle $2 R \cos \theta=m\left(1+R^{2}\right)$ in the plane $\zeta=R \exp i \theta$ orthogonal to the boundary arcs of the Mach circle, without affecting the remaining parts of the region boundary. It is further transformed conformally into the rectangle $0<$ $\sigma<l, 0<\tau<\pi$ in the plane $z=\sigma+i \tau$, with

$$
\sigma_{G}=l, \tau_{G}=\arccos \frac{\sqrt{1-m^{2}} m_{0}-M y_{G}}{\sqrt{1-m^{2}} M-m_{0} y_{G}}
$$

The mapping function, the quantity $l$, the relationship $\theta=\theta(\tau)$ along the shock front image, and the constants $m_{0}$ and $q$ appearing in these are defined in [4] by formulas (3.2)-(3.4).
*) In [4] $h_{v}$ appears with the wrong sign. This did not, however, affect the results.

In the $\zeta$-plane condition (2.2) is expressed in the form

$$
\begin{align*}
& \sqrt{1-m^{2} \sec ^{2} \theta} \frac{\partial p}{\partial n}-(m A \operatorname{tg} \theta-B \operatorname{ctg} \theta) \frac{\partial p}{\partial s}=  \tag{2.5}\\
& \quad(m A \operatorname{tg} \theta-B \operatorname{ctg} \theta) S \delta\left(s-s_{G}\right)
\end{align*}
$$

and in the $z$-plane it can be expressed by

$$
\begin{equation*}
b(\tau)(\partial p / \partial \sigma)-\partial p / \partial \tau=S \delta\left(\tau-\tau_{G}\right) \tag{2.6}
\end{equation*}
$$

where $b(\tau)$ is defined by formula (3.5) in [4].
8. Solution of the boundery value problem. Formula (2.6) along the shock front image and the conditions along other sections of the contour defined at the end of Sect. 1 make it possible to determine function $\Gamma=\partial p / \partial \sigma-i \partial p / \partial \tau$, which is analytic in the rectangle $0<\sigma<l, \quad 0<\tau<\pi$, as the solution of the RiemannHilbert problem [4, 12, 13]

$$
\begin{equation*}
\Gamma(z)=\Phi(z)\left\{c_{0} \frac{\omega^{2}(z)+1}{\omega(z)-\xi\left(l+i \tau_{G}\right)}+c\left[\omega(z)-\xi_{0}\left(z_{0}\right)\right]\right\} \tag{3.1}
\end{equation*}
$$

which along the contour becomes

$$
\begin{equation*}
\Gamma^{-}(z)=\Phi^{-}(z)\left\{c_{0} \frac{\xi^{2}(z)+1}{\xi(z)-\xi\left(l+i \tau_{G}\right)}+c\left[\xi(z)-\xi_{0}\left(z_{0}\right)\right]\right\}+\frac{S \delta\left(\tau-\tau_{G}\right)}{b\left(\tau_{G}\right)-i} \tag{3.2}
\end{equation*}
$$

where the constant $c_{0}$ is defined by the expression

$$
\begin{align*}
& c_{0}=(-1)^{j} S\left|\xi_{\tau}^{\prime}\left(l+i \tau_{G}\right)\right|\left[b^{2}\left(\tau_{G}^{\prime}\right)+1\right]-1 / 2\left\{\pi L_{1}(l+\right.  \tag{3.3}\\
& \\
& \left.\left.i \tau_{G}\right) L_{2}\left(l+i \tau_{G}\right)\left|\Lambda\left(l+i \tau_{G}\right)\right|\left[\xi^{2}\left(l+i \tau_{G}\right)+1\right]\right\}^{-1} \\
& : j=\left\{\begin{array}{l}
0,0<\tau<\tau_{1} \\
1, \tau_{1}<\tau<\tau_{2}, \quad \cos \tau_{1,2}=\frac{m_{0}{ }^{2} \sqrt{m A} \mp \operatorname{tg} \gamma M^{2} \sqrt{B}}{m_{0} M(\sqrt{m A} \mp \operatorname{tg} \gamma \sqrt{B})} \\
2, \tau_{2}<\tau<\pi
\end{array}\right.  \tag{3.4}\\
& \xi_{\tau^{\prime}}\left(l+i \tau_{G}\right)=\frac{2 K}{\pi \sqrt{k}} \frac{\boldsymbol{\vartheta}_{1}\left(\tau_{G}, q\right)}{\vartheta_{4}\left(\tau_{G}, q\right)}\left[k-\frac{\left.\boldsymbol{\vartheta}_{a^{2}\left(\tau_{G}, q\right)}^{\boldsymbol{\vartheta}_{2}{ }^{2}\left(\tau_{G}, q\right)}\right]}{}\right.
\end{align*}
$$

Functions $L_{1}(z), L_{2}(z), \Lambda(z), \omega(z)=\xi+i \eta$ and $\Phi(z)$ along all parts of the region boundary, constants $k, K$ and others appearing in these are defined in Sect. 4 and the beginning of Sect. 5 of [4]; the quantities $\hat{\vartheta}_{1}-\vartheta_{4}$ are elliptic theta functions [14].

The separation of real and imaginary parts of expression (3.2) yields derivatives of pressure perturbation. Along the shock front image we have

$$
\begin{equation*}
\frac{\partial p}{\partial \tau}=\frac{(-1)^{j} L_{1} L_{2}|\Lambda| b(\tau)}{\sqrt{b^{2}(\tau)+1}}\left[c_{0} \frac{\xi^{2}+1}{\xi-\xi_{G}}+c\left(\xi-\xi_{0}\right)\right]-\frac{S \delta\left(\tau-\tau_{G}\right)}{b^{2}(\tau)+1} \tag{3.5}
\end{equation*}
$$

where functions $L_{1}, L_{2},|\Lambda|$ and $\xi$ have $l+i \tau$ as their argument. Here and subsequently $\xi\left(l+i \tau_{G}\right)$ is denoted by $\xi_{G}$. In the expression for the derivative along the wall image

$$
\begin{equation*}
\frac{\partial p}{\partial \tau}=\sqrt{k} \frac{\theta_{3}(\tau, q)}{\theta_{4}(\tau, q)} L_{2} \Lambda\left[c\left(\xi-\xi_{0}\right)+c_{0} \frac{\xi^{2}+1}{\xi-\xi_{G}}\right] \tag{3.6}
\end{equation*}
$$

the argument of these functions is $i \tau$.

The normalization conditions under which constants $c$ and $\xi_{0}$ used in the derivation of the final solution of the problem

$$
\begin{equation*}
B \int_{0}^{\pi} \frac{\partial p}{\partial \tau} \frac{d \tau}{y(\tau)}=v_{5}+\frac{h_{p} B}{y_{G}}-h_{v}, \int_{0}^{\pi} \frac{\partial p}{\partial \tau} d \tau=p_{5} \tag{3.7}
\end{equation*}
$$

are determined and not much different from those in the problem with regular interaction [4]. It can be seen, however, from (3.5) that the integrals which appear there in this problem are singular, hence by Cauchy's definition, only their principal parts can be used as normalizing tools. Taking the above into consideration, it is possible to define constants $c$ and $\xi_{0}$ by formulas (5.5) in [4], where the integrals $I_{1}-I_{6}$ and function $\Psi(\tau)$ are also defined (*). In this case.constants $c_{1}, c_{2}$ and $p_{5}$ are of the form

$$
\begin{align*}
& c_{1}=-\sqrt{1-m^{2}}\left[\left(h_{p}+\frac{S}{b^{2}\left(\tau_{G}\right)+1}\right) y_{G}^{-1}-\frac{2\left(M_{\mathrm{c}}{ }^{2}+1\right) \delta_{1}}{(\chi+1) A M_{c}}\right]  \tag{3.8}\\
& c_{2}=-p_{5}-\frac{S}{b^{2}\left(\tau_{G}\right)+1}, p_{5}=\frac{1 \delta_{1}}{\chi+1} \frac{M_{c}{ }^{2}+1}{2 A M_{c}} M \cos \gamma+h_{p}
\end{align*}
$$

where $\delta_{1}$ is the change of the compression shock angle of inclination after its encounter with the incident wave. The principal values of integrals $I_{5}$ and $I_{6}$ are taken here, while for $\tau_{G} \rightarrow 0$ and $\pi$ they exist in the conventional meaning, since then $b(\tau) \rightarrow 0$ at the rate of a linear function.
4. Shape of the compression shock distorted section. The second of Eqs. (2.1) for the boundary value of the derivative of velocity perturbation along the shock front can be solved with respect to the derivative $f^{\prime}$ of its shape. After differentiation with respect to $y$, the derivative $\partial v / \partial y$ defined by (2.3) appears in the righthand side of the derived expression. We obtain the differential equation

$$
\begin{equation*}
f_{y y}^{\prime \prime}=-\frac{x+1}{2 M_{c}} \frac{a_{1}}{a_{\propto}} \frac{B}{1-M_{c}^{2}} \frac{1}{y}\left[\frac{\partial p}{\partial y}-h_{p} \delta\left(y-y_{G}\right)\right] \tag{4.1}
\end{equation*}
$$

Integration with allowance for the boundary condition $f=f_{y}{ }^{\prime}=0$ at point $A$ yields the expression

$$
f(y)=-\frac{x+1}{2 M_{\mathrm{c}}} \frac{a_{\mathrm{t}}}{a_{\infty}} \frac{B}{1-M_{\mathrm{c}}^{2}} \int_{0}^{\tau(y)}[y-t(\lambda)]\left[\frac{\partial y}{\partial \lambda} \cdots h_{p} \delta\left(t-t_{G}\right)\right] \frac{d \lambda}{t(\lambda)}
$$

Symbols $y(\tau)$ and $t(\lambda)$ define one and the same dependence. The normalization conditions (3.7) specify the observance of boundary conditions for functions $f$ and $f_{y}{ }^{\prime}$ at point $C$.
5. Nature of solution in the neighborhood of the triple point and passing to limit of the diffraction problem with regular interaction. All components of the arising singularity are represented by the derivative $\partial p / \partial \tau$ along the shock front image (3.5). The solution consists of a smooth function, a logarithmic singularity, and a finite compression shock $-S /\left[b^{2}\left(\tau_{C}\right)+1\right]$. With the use of (2.4) and (1.1) it is possible to indicate the interdependence of input parameters

[^0]which satisfies condition $s=0$. This interdependence is shown in Fig. 2 by thin continuous lines. In accordance with (3.3) we then have $c_{0}=0$, hence neither a compression shock nor a logarithmic singularity are present. In a reasonably small neighborhood of the triple point the shock fronts and the tangential discontinuity separate streams which may differ as little as desired from a uniform pattern. The indicated bifurcation curves exactly correspond to those presented in $[10,11]$. The outer and middle posi tions of the triple point $y_{G}=\mp \sqrt{1-m^{2}}, 0$ (continuous heavy solid lines in Fig. 2) impart indeterminacy to formula (3.5) for $\tau \rightarrow \tau_{G}=0$, $\pi$, or $\operatorname{arc} \cos \left(m_{0} / M\right)$ since then $b(\tau) \rightarrow 0$. These indeterminacies are readily expanded by the substitution $\xi_{G}-\xi=\xi_{\tau}^{\prime}\left(\tau_{G}-\tau\right)$ since $\xi_{\tau}{ }^{\prime}$ appears as a factor in $c_{0}$. In the neighborhood of point $\tau=\arccos \left(m_{0} / M\right)$ we then have
$$
\partial p / \partial y=2 S m / \pi \sqrt{1-m^{2}}-S \delta\left(y-y_{G}\right), S=-h_{p}
$$

At the extreme points the delta function of the same sign and with coefficient $S$ (which assumes other values) is combined with other, also finite quantities, so that for $\tau_{G} \rightarrow \arccos \left(m_{0} / M\right), 0$, or $\pi$ the dependence of pressure on the coordinate along the shock front is free of logarithmic singularities (points of bifurcation of the second kind). Thus, if in a specific problem by varying any of the external parameters (e.g. angle $\chi$ ) point $G$ is made to tend to point $A$ or $C\left(y_{G} \rightarrow \mp V 1-m^{2}, \tau_{G} \rightarrow 0\right.$, $\pi$ ), the logarithmic singularity is apparent in an ever diminishing neighborhood of point $G$ and is levelled out; at the limit only a finite pressure jump of magnitude $S$ remains at the triple point.

It should be noted that the derivative $\partial p / \partial y$ remains infinite also at the limit, because $\partial \tau / \partial y \rightarrow \infty$. when $y \rightarrow+\sqrt{1-m^{2}}$, although the limit value of $\partial p / \partial \tau$ is finite, i. e. for $y_{G}= \pm \sqrt{1-m^{2}}$, curves $p=p(y)$ have vertical tangents.
It can be readily shown by substituting $y_{G}=-\sqrt{1-m^{2}}$, into formula (2.4) that the pressure jump, which is equal $S$, at the shock front image for the extreme righthand position of point $G$ on the latter is equal $p_{4}$ which exists to the right of point $A$ in the solution of the problem of diffraction with regular interaction [4], when point $G$ approaches point $A$ along the Mach arc $A F$. Formulas ( 3.8 ) for constants $c_{1}$ and $c_{2}$ are the same as the formulas in Sect. 5 of [4]. This settles the question of the continuity of transition of the derived solution for the pressure into the solution of the diffraction problem with regular interaction.

Of interest are also points $\tau_{G}=\tau_{1,2}, y_{G}=\mp \sqrt{m B / A}$
when $b(\tau) \rightarrow \infty$, the finite pressure jump vanishes, and the logarithmic singularity is symmetric (bifurcation of the third kind). These cases are represented in Fig. 2 by dashed lines.

Equation (4.1) implies that the angle of inclination $f_{y}{ }^{\prime}$ of the front is defined by functions of the same kind as the pressure perturbation. In this case the summary coeffiicient at the delta function in accordance with (3.5) and (4.1) is equal to

$$
\begin{equation*}
B\left[h_{p}+S /\left(b^{2}+1\right)\right] / M_{1} y_{G} \tag{5,1}
\end{equation*}
$$

It relates to every position of point $G$ a certain magnitude of the shock front angle of inclination, although the second term in expression (3.5) which appears in (4.1) indicates that the tangent of that angle tends to infinity when $y \rightarrow y_{G}$. This and the existence of the logarithmic singularity of pressure show the inadequacy of the linear analy-
sis of this problem. The shape of the compression shock in the small neighborhood of point $G$ is complex. When considering the shape of front curves calculated and presented below, and when attempting to obtain triple shock configurations by geometrical construction in the neighborhood of point $G$, it must be borne in mind that the boundary conditions of the problem are related here to the line of the unperturbed oblique shock and that point $G$ belongs to that line.

With allowance for the pointed out defect the obtained jump of the angle of inclination of the front can be considered related to the boundaries of some neighborhood of point $G$.
In the case of bifurcation for $S=0$ branches of the distorted shock at the triple point are at the angle $\pi-h_{p} B / M_{1} y_{G}$ to each other, and the described defect disappears. It is also absent, when point $G$ coincides with points $A$ or $C$, where the inclination of the front outside the limits of the diffraction region is defined by the expression (5.1). It can be shown by substituting $y_{G}=-\sqrt{1-m^{2}}$ into (5.1) that at point $A$ the inclination is

$$
\delta_{2}=B\left(p_{4}-h_{p}\right) / M_{1} \sqrt{1-m^{2}}
$$

which can also be obtained from formula (2.2) in [4] for the refraction angle of the shock front in the solution of the diffraction problem with regular interaction.
6. Retults of computations. The properties of solutions are illustrated in Fig. 3, where pressure distribution curves along the shock front ( $a, b$ ) of the shock front shape (c), and of pressure at the wall (d) are shown. These curves relate to the motion of the wedge with a $20^{\circ}$ vertex angle at $M_{\infty}=3.2$ and 11 values of angle $\chi$ in the considered range $\left\langle 125.8^{\circ}, 132.1^{\circ}, 138.6^{\circ}, 145.3^{\circ}, 151.8^{\circ}, 158.1^{\circ}, 164.1^{\circ}, 169.5^{\circ}, 174.3^{\circ}, 178.5^{\circ}\right.$ and $182.2^{\circ}$ ). This range is fixed by the curves shown in Fig. 2 ; the angle of inclination of the compression shock $\alpha=36.3^{\circ}$ and the pressure ratio $p_{\infty} / p_{1}=0.248$.

To unify scales the pressure along the front and the shape of the latter are represented in terms of coordinate $y^{*}=\dot{y} / \sqrt{1-m^{2}}$ which varies from -1 (at point $A$ ) to +1 (at point $C$ ), and the pressure at the wall is given in terms of coordinate $r$ measured along the wall in both directions from point $E$ so that $r=1$ at points $A$ and $C$.

The chosen angles $\chi$ correspond to the spacing of point $G$ at $0.2 \sqrt{1-m^{2}}$, with angle $\chi=125.8^{\circ}$ corresponding to the extreme right-hand position of point $G$ at $y_{C_{7}}{ }^{*}$ --1 , and subsequently, in the enumeration order, to positions of point $G$ shifted to the left at the indicated pitch. This makes it easy to select the required curve of the set, since each curve of pressure at the front has a singularity above point $G$, and the shape of the front in the neighborhood of that point has its maximum deviation from the initial straight line. The position of wall pressure curves relates to above values of angle $\chi$ with the upper curve corresponding to angle $\chi=125.8^{\circ}$.

The increase of pressure at the wall in the left-hand corner of the diffraction region at considerable angles $\chi$ is explained by the proximity of point $G$ to that part of the wall, which in such diffraction modes lies on the oblique section of the jump which is close to the wedge edge.
The curves of pressure along the distorted part of the shock front (curves (a)) in the proximity of the boundary points $A$ and $C$ do not show the properties of function $p(y)$ which were described in Sect. 5. The tendency of this function to the ultimate limit for $y^{*} \rightarrow \pm 1$ in the cases of $y_{G}{ }^{*}= \pm 1$ becomes apparent only in a very small neighborhood of these points. This is shown by curves (b) drawn in large scale in the nighborhood
of point $A$ with related values of $y_{G}{ }^{*}$ indicated under the dash-dot lines.
If point $G$ coincides with point $A$, the pressure in its proximity falls with increasing $y$ according to a near-logarithmic law, Its shift to the left results in the appearance of a logarithmic singularity; the pressure jump changes its sign at transition of point $G$ through the third kind bifur-


Fig. 3 cation point $y_{G}=y\left(\tau_{1}\right)$ and vanishes at that point, while the pressure curve in the neighborhood of point $G$ becomes symmetric. Further shift of point $G$ to the left brings it to the bifurcation point thus ensuring the continuity of pressure along the whole of the shock front distorted section and causing the change of sign of the logarithmicsingularity and of the pressure jump (curves exactly corresponding to bifurcation or to the case of $y_{G}=y\left(\tau_{1}\right)$ do not appear in Fig. 3, but curves close to these are readily discernible). The described singularities reappear when point $G$ moves to the left of the bifurcation point until it reaches the second kind bifurcation point $y_{G}=0$, where the logarithmic singularity again changes its sign and vanishes, while the pressure jump persists up to $y_{G}=y\left(\tau_{2}\right)$. The behavior of function $p(y)$ for $y_{G}{ }^{*}=y^{*}\left(\tau_{2}\right)$ and $y_{G}{ }^{*}=1$ is the same as for $y_{G}{ }^{*}=y^{*}\left(\tau_{1}\right)$ and $y_{G}{ }^{*}=-1$. All computations performed are presented in Figs. 2 and 3 under assumption that $x=1.4$.
7. On the investigations of Inger. The assumption of a wedge of a small opening angle and of its hypersonic speed made it possible for Inger [8,9] to use lighthill's symmetric solution of the problem of shock wave diffraction over a blunt edge, with the addition of only one term to take into account the additional pressure upstream of the diffraction region. The solution derived in this manner is continuous at the point of intersection of fronts.

One of the two constants appearing there and, also, the slope of the tangential discontinuity at the triple point were determined by the author on the assumption of continuity
of the distorted front at that point and normal to the tangential dicontinuity of the velocity projection, obtaining for the tangential discontinuity a direction different by a finite angle from that of the velocity vector of the uniform stream behind the plane shock front, although owing to the problem linearity, this angle can only be of the order of $e$. It can be readily shown that this method violates, for instance, the condition of continuity of pressure at the triple point (by determining the slope of the tangential discontinuity and the conditions for velocities along it, then expressing the projections of these in terms of the front slope and, finally, determining the break in the front and substituting it into the condition for pressure at that discontinuity). Other aspects could also be cited, but there is no need for that. As described above, in the case of sporadic interaction of two shock waves of independent origin moving toward each other from opposite directions, the flow in the small neighborhood of the triple point can be considered as consisting of a system of uniform streams, and the solution for pressure to be continuous at that point only for certain specific flow parameters which satisfy the condition of bifurcation [10, 11], and not for any arbitrary values of these.

The distribution of pressure perturbation along the wall and the shock front was computed in [8]. The preturbation along the wall has a maximum which exceeds its maximum value at the front, a fact which was stressed by the author in the discussion of results,

Since the normal derivative of pressure perturbation along the wall is zero, considerations of symmetry imply that the indicated fact does not conform to the principle of maximum of harmonic function modulus.

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[^0]:    *) The factor $(-1)^{i}$ was inadvertently omitted in the expression for $\Psi(\tau)$ in [4].

